On Cosmic Acceleration without Dark Energy

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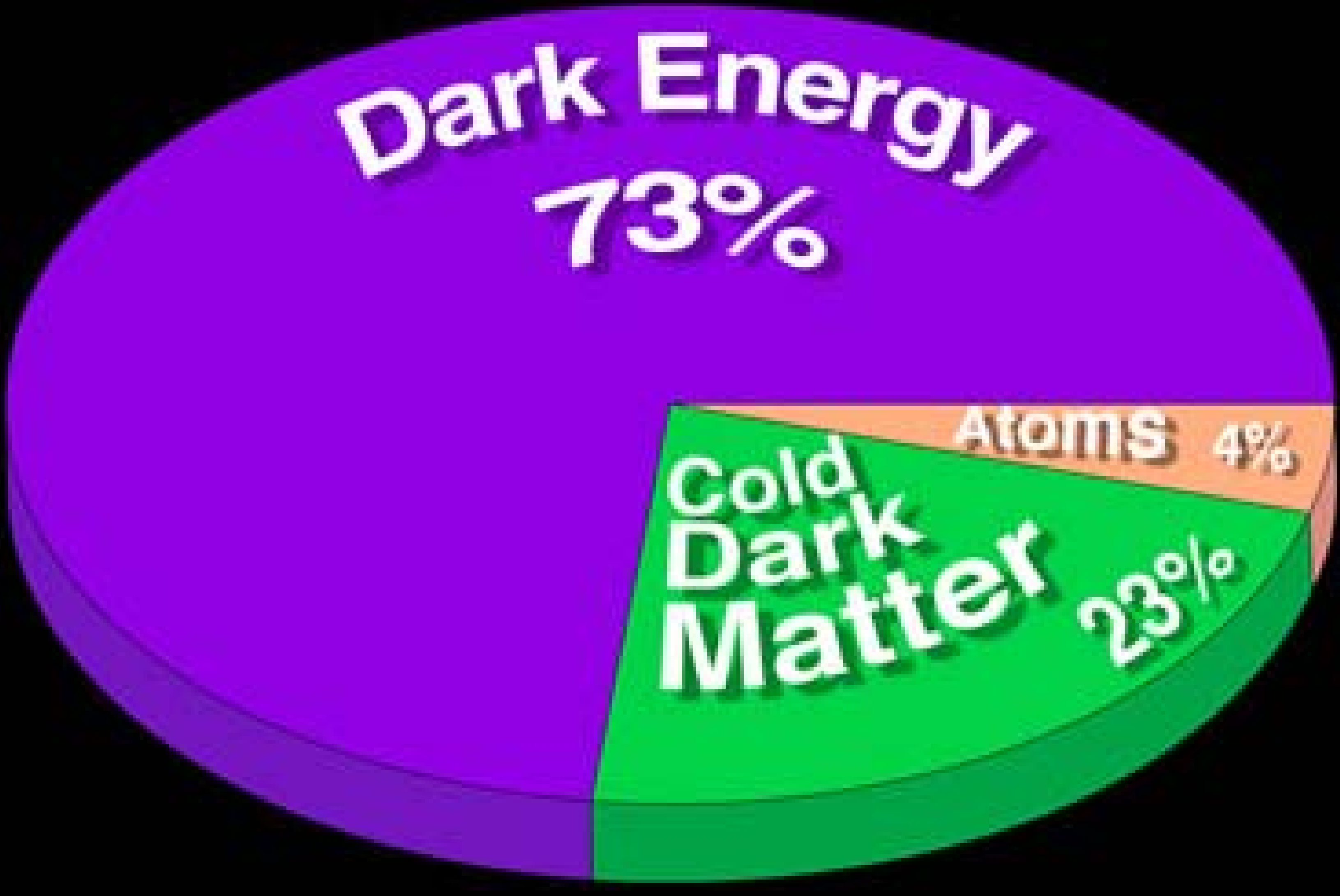
All work is the result of collaborations with Sabino Matarrese and Antonio Riotto (Padova) [and occasionally Alessio Notari (McGill)]

ΛCDM



Do we "know" there is dark energy?

- Assume model cosmology:
  - Friedmann model: $H^2 + k/a^2 = 8\pi G\rho / 3$
  - Energy (and pressure) content: $\rho = \rho_M + \rho_R + \rho_\Lambda + \ldots$
  - Input or integrate over cosmological parameters: H_0, *etc*.

- Calculate observables $d_L(z)$, $d_A(z)$, …

- Compare to observations

- Model cosmology fits with ρ_Λ, but not without ρ_Λ

- All evidence for dark energy is ***indirect*** : observed $H(z)$ is ***not*** described by $H(z)$ calculated from the Einstein-de Sitter model [spatially flat ($k = 0$) from CMB ; matter dominated ($\rho = \rho_M$)]

Take sides!

- Can't hide from the data – ΛCDM too good to ignore
  - SNIa
  - Subtraction: $1.0 - 0.3 = 0.7$
  - Age
  - Large-scale structure
  - …

  } $H(z)$ not given by Einstein–de Sitter

$$3H^2 + k/a^2 \neq 8\pi G\,\rho_{\mathrm{MATTER}} \rightarrow G_{00} \neq 8\pi G\,T_{00}(\text{matter})$$

- Dark energy (modify right-hand side of Einstein equations)
  - "Just" Λ, a cosmological constant
  - If not constant, what drives dynamics (scalar field)
- Gravity (modify left-hand side of Einstein equations)
  - Beyond Einstein (non-GR: branes, *etc*.)
  - (Just) Einstein (GR: back reaction of inhomogeneities)

Modifying the left-hand side

- Braneworld modifies Friedmann equation — Binetruy, Deffayet, Langlois
- Phenomenological approach — Freese & Lewis

$$H^2 = A\rho\left[1+\left(\rho/\rho_{\text{cutoff}}\right)^{n-1}\right]$$

- Gravitational force law modified at large distance — Deffayet, Dvali & Gabadadze

  Five-dimensional at cosmic distances
- Tired gravitons — Gregory, Rubakov & Sibiryakov; Dvali, Gabadadze & Porrati

  Gravitons metastable - leak into bulk
- Gravity repulsive at distance $R \approx \text{Gpc}$ — Csaki, Erlich, Hollowood & Terning
- $n = 1$ KK graviton mode very light, $m \approx (\text{Gpc})^{-1}$ — Kogan, Mouslopoulos, Papazoglou, Ross & Santiago
- Einstein & Hilbert got it wrong — Carroll, Duvvuri, Turner, Trodden

$$S = \left(16\pi G\right)^{-1}\int d^4x\sqrt{-g}\left(R-\mu^4/R\right)$$

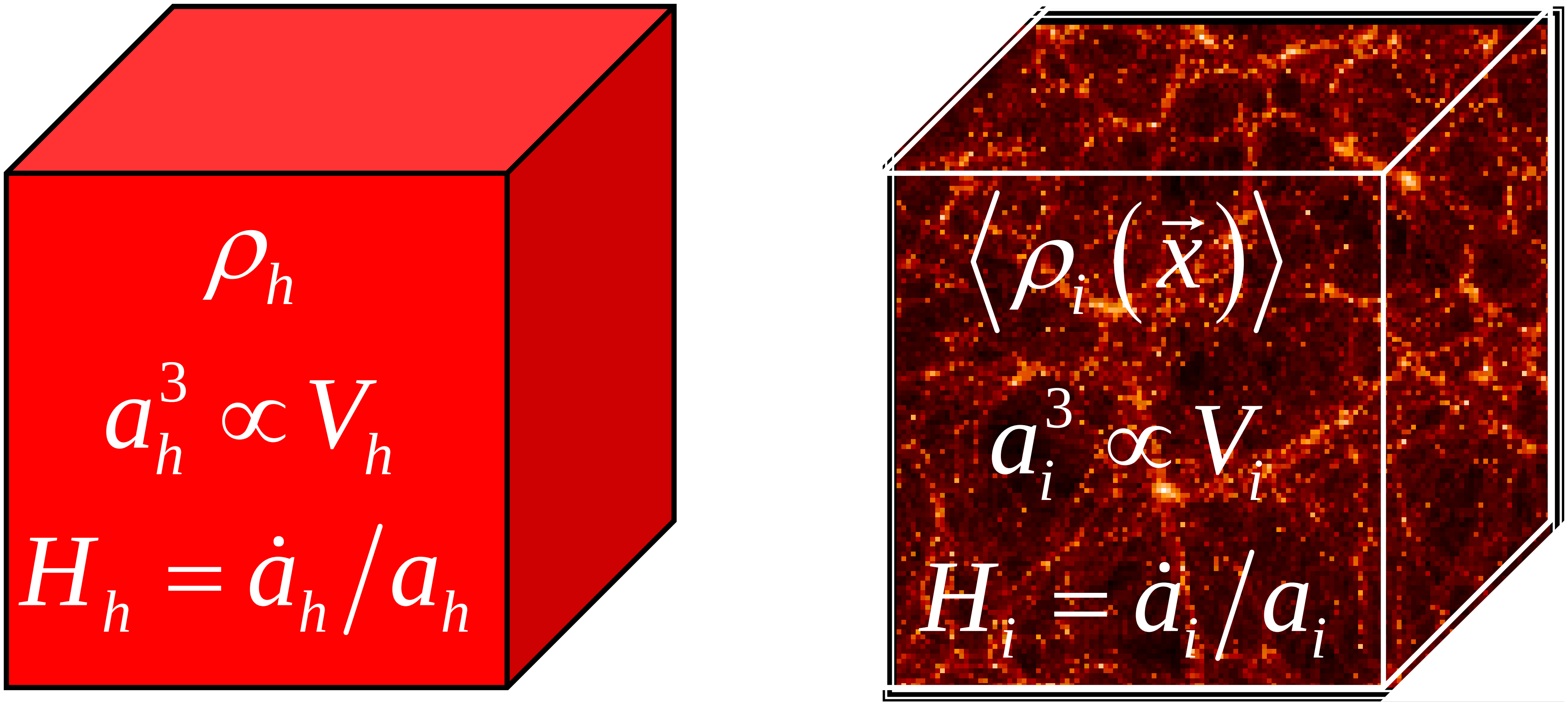
- Backreaction of inhomogeneities — Räsänen; Kolb, Matarrese, Notari & Riotto; Notari; Kolb, Matarrese & Riotto

Acceleration from inhomogeneities

- Most conservative approach — nothing new
  - no new fields (like $10^{-33}\,\mathrm{eV}$ mass scalars)
  - no extra long-range forces
  - no modification of general relativity
  - no modification of gravity at large distances
  - no Lorentz violation
  - no extra dimensions, bulks, branes, etc.
  - no faith-based (anthropic/landscape) reasoning
- Magnitude?: calculable from observables related to $\delta\rho/\rho$
- Why now?: acceleration triggered by era of non-linear structure

Acceleration from inhomogeneities

Homogeneous model | Inhomogeneous model



$$\rho_h = \langle \rho_i(\vec{x}) \rangle \Rightarrow H_h = H_i \ ?$$

We think not!

Acceleration from inhomogeneities

- View scale factor as zero-momentum mode of gravitational field
- In homogeneous/isotropic model it is the only degree of freedom
- Inhomogeneities: non-zero modes of gravitational field
- Non-zero modes interact with and modify zero-momentum mode

Cosmology $\leftrightarrow$ scalar field theory analogue

	cosmology	scalar-field theory
zero-mode	a	$\langle\phi\rangle$ (vev of a scalar field)
non-zero modes	inhomogeneities	thermal/finite-density bkgd.
physical effect	modify $a(t)$ *e.g.*, acceleration	modify $\langle\phi(t)\rangle$ *e.g.*, phase transitions

Different approaches

Standard approach	Our approach
• Model an inhomogeneous Universe as a homogeneous Universe model with $\rho = \langle \rho \rangle$	• Expansion rate of an inhomogeneous Universe $\neq$ expansion rate of homogeneous Universe with $\rho = \langle \rho \rangle$
• Zero mode [$a(t)$ / $V^{1/3}$] is the zero mode of a homogeneous model with $\rho = \langle \rho \rangle$	• Inhomogeneities modify zero-mode [effective scale factor is $a_D \equiv V_D^{1/3}$]
• Inhomogeneities only have a local effect on observables	• Effective scale factor has a (global) effect on observables
• Cannot account for observed acceleration	• Potentially can account for acceleration without dark energy or modified GR

Acceleration from inhomogeneities

- We *do not* use super-Hubble modes for acceleration.
- We *do not* depend on large gravitational potentials such as black holes and neutron stars.
- We assert that the back reaction should be calculated in a frame comoving with the matter—other frames can give spurious results.
- We demonstrate large corrections in the gradient expansion, but the gradient expansion technique can not be used for the final answer—so we have indications (not proof) of a large effect.
- The basic idea is that small-scale inhomogeneities “renormalize” the large-scale properties.

Inhomogeneities–cosmology

- Our Universe is inhomogeneous

- Can define an average density $\langle \rho \rangle$
- The expansion rate of an *inhomogeneous* universe of average density $\langle \rho \rangle$ is *NOT!* the same as the expansion rate of a *homogeneous* universe of average density $\langle \rho \rangle$!
- Difference is a new term that enters an effective Friedmann equation — the new term need not satisfy energy conditions!
- We deduce dark energy because we are comparing to the wrong model universe (*i.e.,* a homogeneous/isotropic model)

Ellis, Barausse, Buchert, Ellis, Kolb, Matarrese, Notari, Räsänen, Riotto, Schwarz

Inhomogeneities–example

Kolb, Matarrese, Notari & Riotto

- Perturbed Friedmann–Lemaître–Robertson–Walker model:

$$G_{\mu\nu}\left(\vec{x},t\right) = G_{\mu\nu}^{\mathrm{FLRW}}\left(t\right) + \delta G_{\mu\nu}\left(\vec{x},t\right)$$

$$G_{00}^{\mathrm{FLRW}}\left(t\right) + \delta G_{00}\left(\vec{x},t\right) = 8\pi G T_{00}\left(\vec{x},t\right)$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left[\left\langle\rho\right\rangle - \frac{3}{8\pi G}\left\langle\delta G_{00}\right\rangle\right]$$

- $(\dot{a}/a)^2$ *is not* $8\pi G\langle\rho\rangle/3$

- ($\dot{a}/a$ *is not even the expansion rate*)

- Could $\langle\delta G_{00}\rangle$ play the role of dark energy?

Inhomogeneities–cosmology

- For a general fluid, four velocity $u^\mu = (1,\vec{0})$
  (local observer comoving with energy flow)

- For irrotational dust, work in synchronous and comoving gauge

$$ds^2 = -dt^2 + h_{ij}(\vec{x},t)dx^i dx^j$$

- Velocity gradient tensor

$$\Theta^i{}_j = u^i{}_{;j} = \tfrac{1}{2}h^{ik}\dot{h}_{kj} = \Theta\delta^i{}_j + \sigma^i{}_j \qquad (\sigma^i{}_j \text{ is traceless})$$

- Θ is the volume-expansion factor and $\sigma^i{}_j$ is the shear
  (shear will have to be small)

- For flat FLRW, $h_{ij}(t) = a^2(t)\delta_{ij}$
  $\Theta = 3H$ and $\sigma^i{}_j = 0$

Inhomogeneities and acceleration

- Local deceleration parameter positive:

Hirata & Seljak; Flanagan; Giovannini; Alnes, Amarzguioui & Gron

$$q = -\frac{\left(3\dot{\Theta} + \Theta^2\right)}{\Theta^2} = 6\left(\sigma^2 + 2\pi G\rho\right) \geq 0$$

- However must coarse-grain over some finite domain:

$$\left\langle \Theta \right\rangle_D = \frac{\int_D \sqrt{h}\, \Theta\, d^3x}{\int_D \sqrt{h}\ d^3x}$$

- Evolution and smoothing do not commute:

Buchert & Ellis; Kolb, Matarrese & Riotto

$$\left\langle \Theta \right\rangle_D^{\bullet} = \left\langle \Theta^{\bullet} \right\rangle_D + \left\langle \Theta^2 \right\rangle_D - \left\langle \Theta \right\rangle_D^2 \geq \left\langle \Theta^{\bullet} \right\rangle_D$$

- $\left\langle \Theta \right\rangle_D^{\bullet} \neq \left\langle \Theta^{\bullet} \right\rangle_D$ although $\left\langle \Theta^{\bullet} \right\rangle_D$ can't accelerate, $\left\langle \Theta \right\rangle_D^{\bullet}$ can!

Inhomogeneities and smoothing

Kolb, Matarrese & Riotto astro-ph/0506534; Buchert & Ellis

- Define a coarse-grained scale factor:

$$a_D \equiv \left(V_D / V_{D0}\right)^{1/3} \qquad V_D = \int_D d^3x \sqrt{h}$$

- Coarse-grained Hubble rate:

$$H_D = \frac{\dot{a}_D}{a_D} = \tfrac{1}{3}\langle\Theta\rangle_D$$

- Effective evolution equations:

$$\frac{\ddot{a}_D}{a_D} = -\frac{4\pi G}{3}\left(\rho_{\rm eff} + 3p_{\rm eff}\right) \qquad \rho_{\rm eff} = \langle\rho\rangle_D - \frac{Q_D}{16\pi G} - \frac{\langle R\rangle_D}{16\pi G}$$

$$\left(\frac{\dot{a}_D}{a_D}\right)^2 = \frac{8\pi G}{3}\rho_{\rm eff} \qquad 3p_{\rm eff} = -\frac{3Q_D}{16\pi G} + \frac{\langle R\rangle_D}{16\pi G}$$

not described by a simple $p = w\,\rho$

- Kinematical back reaction: $Q_D = \tfrac{2}{3}\left(\langle\Theta^2\rangle_D - \langle\Theta\rangle_D^2\right) - 2\langle\sigma^2\rangle_D$

Inhomogeneities and smoothing

- Kinematical back reaction: $$Q_D = \tfrac{2}{3}\left(\left\langle \Theta^2 \right\rangle_D - \left\langle \Theta \right\rangle_D^2\right) - 2\left\langle \sigma^2 \right\rangle_D$$

- For acceleration: $$\rho_{\text{eff}} + 3p_{\text{eff}} = \left\langle \rho \right\rangle_D - \frac{Q_D}{4\pi G} < 0$$

- Integrability condition (GR): $$\left(a_D^6 Q_D\right)^{\bullet} + a_D^4 \left(a_D^2 \left\langle R \right\rangle_D\right)^{\bullet} = 0$$

- Acceleration is a pure GR effect:
  - curvature vanishes in Newtonian limit
  - Q_D will be exactly a pure boundary term, and small

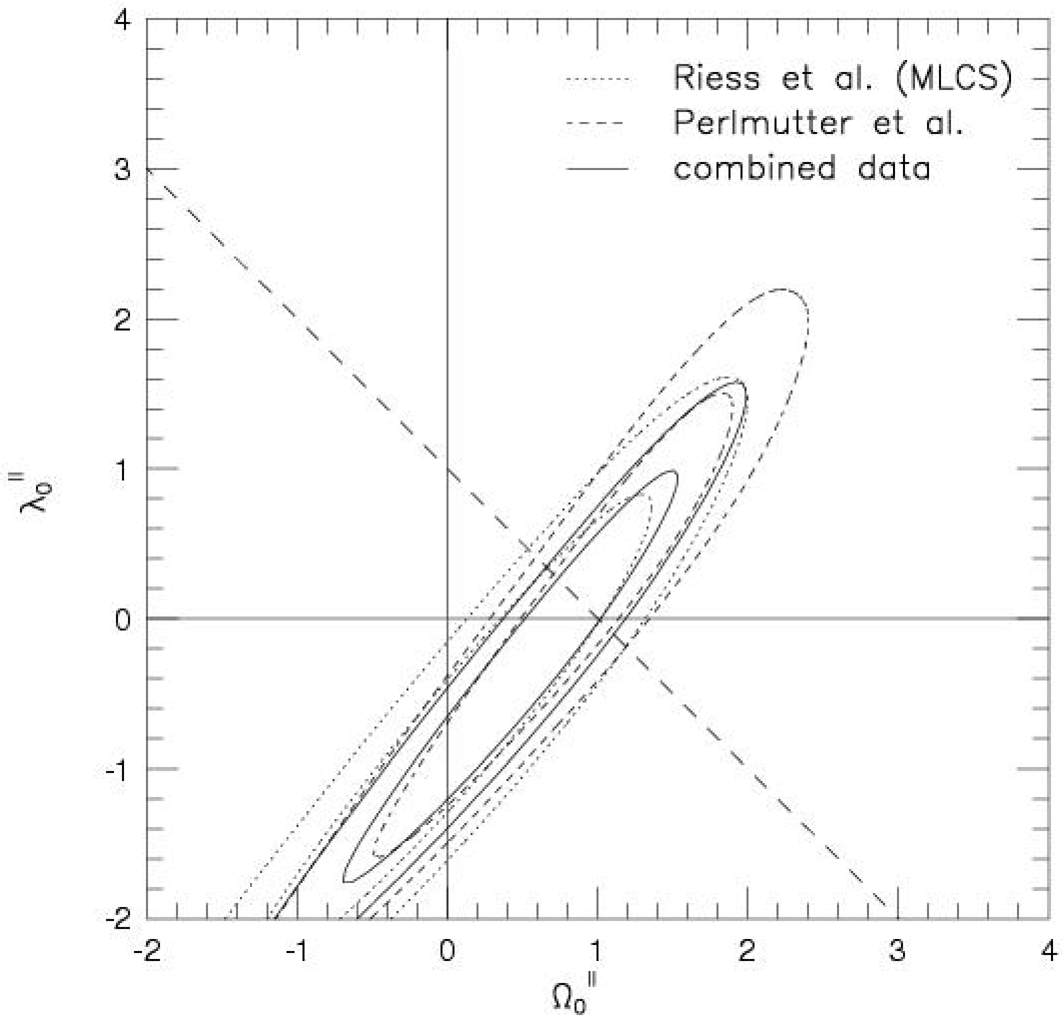
- Particular solution: $3Q_D = -\mathrm{h}R\mathrm{i}_D = \text{const.}$
  - *i.e.*, $\Lambda_{\text{eff}} = Q_D$ (so Q_D acts as a cosmological constant)

Inhomogeneities

- Does this have anything to do with our universe?
- Have to go to non-perturbative limit!
- How to relate observables ($d_L(z)$, $d_A(z)$, $H(z)$, …) to Q_D & $\langle R \rangle_D$?
- Can one have large effect and isotropic expansion/acceleration? (*i.e.,* will the shear be small?)
- What about gravitational instability?
- Toy model proof of principle: Tolman-Bondi dust model
  Nambu & Tanimoto; Moffat; Tomita, …

Observational consequences

- Spherical model
- Overall Einstein–de Sitter
- Inner underdense 200 Mpc region
- Compensating high-density shell
- Calculate $d_L(z)$
- Compare to SNIa data
- Fit with $\Lambda = 0$!



Tomita, 2001

Comments

- *"Do you believe?"* is not the relevant question
- Acceleration of the Universe is important; this must be explored
- How it could go badly wrong:
  - Backreaction should not be calculated in frame comoving with matter flow
  - Series re-sums to something harmless
  - No reason to stop at first large term
  - Synchronous gauge is tricky
    - ☹ Residual gauge artifacts
    - ☹ Synchronous gauge develops coordinate singularities at late time (shell crossings)
    - ☺ Problem could be done in Poisson gauge

Conclusions

- Must properly smooth inhomogeneous Universe
- In principle, acceleration possible even if "locally" $\rho + 3p > 0$
- Super-Hubble modes, of and by themselves, cannot accelerate
- Sub-Hubble modes have large terms in gradient expansion
  - Newtonian terms can be large but combine as surface terms
  - Post-Newtonian terms are not surface terms, but small
  - Mixed Newtonian £ Post-Newtonian terms can be large
  - Effect from "mildly" non-linear scales
- The first large term yields effective cosmological constant
- No reason to stop at first large term
- Can have $w < -1$?
- Advantages to scenario:
  - No new physics
  - "Why now" due to onset of non-linear era

Many issues:

- non-perturbative nature
- shell crossing
- comparison to observed LSS
- gauge/frame choices
- physical meaning of coarse graining

Program:

- can inhomogeneities change effective zero mode?
- how does (does it?) affect observables?
- can one design an inhomogeneous universe that accelerates?
- could it lead to an apparent dark energy?
- can it be reached via evolution from usual initial conditions?
- does it at all resemble our universe?
- large perturbative terms resum to something harmless?

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